Matlab-based Voltage Stability Bifurcation Analysis Toolbox

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Introduction

Voltage Stability Toolbox (VST) developed by CEPE (Center for Electric Power Engineering), Drexel University in 1995.

can be used to analyze voltage stability bifurcation problem and provide intuitive information for power system planning, operation, and control.

Fig.1 The view of VST in Matlab
Introduction

Voltage Stability Toolbox (VST)

Fig.2 Schematic diagram of VST
**Introduction**

Motivation

- An important theory tools in voltage stability research
- Accurate in analyzing system stability around critical point neighborhood
- Modeling difficulties
- Complex calculations
- Auto 97

Bifurcation

VST
Introduction

Motivation

Bifurcation theory of VST

Differential algebraic equations (DAE)

\[
\begin{align*}
\dot{x} &= f(x, y, \mu) \\
0 &= g(x, y, \mu)
\end{align*}
\] (1)

where, \( x \in \mathbb{R}^n \) is differential state variable, \( y \in \mathbb{R}^m \) is algebraic state variable, \( \mu \in \mathbb{R}^k \) is system control parameter, \( n, m \) and \( k \) are the corresponding dimensions.
2 Bifurcation theory of VST

Static bifurcation

\[ \mu = \mu^0 + \alpha \mu' \]  

(2)

where, \( \mu^0 \) is the initial real or reactive power,

\( \alpha \) is the bifurcation parameter which denotes the level of load,

\( \mu' = [d_{P_e}^T \ d_{R_i}^T \ d_{Q_1}^T]^T \) is the vectors of searching directions,

\( T \) denotes transpose, elements of \( \mu' \):

\[
\begin{align*}
    d_{P_e} & = [d_{P_1} \ ... \ d_{P_{n_e}}]^T \\
    d_{R_i} & = [d_{P_{n_e}+1} \ ... \ d_{P_{n_e+n_{pq}}}]^T \\
    d_{Q_1} & = [d_{P_{n_e}+1} \ ... \ d_{P_{n_e+n_{pq}}}]^T
\end{align*}
\]
Bifurcation theory of VST

Static bifurcation

\[
\begin{align*}
F(z, \mu) &= 0 \\
F_z(z, \mu)v &= 0 \text{ or } w^TF_z(z, \mu) = 0 \\
F_\mu(z, \mu)v &= 1 \text{ or } w^TF_\mu(z, \mu) = 1
\end{align*}
\]

(3)

where, \( F(z, \mu) = [f^T(x, y, \mu) \quad g^T(x, y, \mu)]^T = 0 \) is system equation, \( z = [x^T \quad y^T]^T \) is system state variable, \( v, w \) are the corresponding right and left eigenvector of Jacobi zero eigenvalue.
Bifurcation theory of VST

Dynamic bifurcation

Linearization equation (1) at equilibrium point:

\[(x, y, \mu) \text{ (which satisfies } f(x, y, \mu) = 0 \text{ and } g(x, y, \mu) = 0)\]

\[E = \{(x, y, \mu) \in \mathbb{R}^{n+m+k} | f(x, y, \mu) = 0, g(x, y, \mu) = 0\} \quad (4)\]

If \(g_y(x, y, \mu)\) is nonsingular

Linearization equation (1):

\[\dot{x} = h(x, \mu) = f(x, \psi(x, \mu), \mu) \quad (5)\]

where, \(\psi(x, \mu)\) is the function of equilibrium point, which satisfies the equation \(g[x, \psi(x, \mu), \mu] = 0, y\) is \(\psi(x, \mu)\).
Bifurcation theory of VST

Dynamic bifurcation

Set:

\[ J = f_x - f_y g_y^{-1} g_x \]  \hspace{1cm} (6)

When equation (1) at equilibrium point \((x_0, y_0, \mu_0)\) satisfies:

\[
\begin{align*}
\det[g_y(x_0, y_0, \mu_0)] &= 0 \\
\text{tr}[f_y A(g_y) g_x] &= 0 \\
\det \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} &\neq 0 \\
\det[g_y(x_0, y_0, \mu_0)]_{\mu} &\neq 0
\end{align*}
\]  \hspace{1cm} (7)

where, \(\det\) denotes determinant of matrix, \(\text{tr}\) denotes determinant trace, \(\mu\) denotes the Adjoint matrix.
3 Simulation Results

Fig. 3 Five-bus system with three generators

Fig. 4 IEEE 30-bus test system
Simulation Results

Fig. 5 Voltage magnitude at bus 4 of static bifurcation (five-bus system)

Fig. 6 Voltage magnitude at bus 4 of dynamic bifurcation and singular points (five-bus system)
3 Simulation Results

Fig. 7 Critical eigenvalues plot of the system matrix (five-bus system)

Fig. 8 The right and left eigenvectors corresponding to the zero eigenvalue of the system matrix (IEEE 30 bus system)
Conclusions

(1) Open source code and user-friendly graphical interface

(2) Very helpful to understand voltage stability and nonlinear bifurcation phenomena

(3) Feasibility and validity
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