Mutual Coupling Modeling in Transmission Lines directly in the Phase Domain

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Objectives:

- Mutual coupling between phases using only lumped elements (without the complex use of electromagnetic field theory).
- Simplified digital model based on line parameters (without the explicit use of modal transformations between the phase and mode domains).
- Easy interaction between the line model with time-variable and non-linear components (capacitor bank, corona, variable load profile, etc).
- Simplified application for didactic or research purposes.
Theoretical Features

\[ I_{in} = V_{in} Y \\]
\[ I_{out} = V_{out} Y \\]

\[ IVZ_{\text{inbound}} c\text{oth}(\gamma) \ell\text{sch}(\gamma) \]
\[ IVZ_{\text{outbound}} c\text{sch}(\gamma) \ell\text{oth}(\gamma) \]
Theoretical Features

\[
\begin{align*}
\frac{di_1(t)}{dt} & = i_1(t) - i_2(t) + \frac{1}{C} v_1(t) \\
\frac{di_2(t)}{dt} & = i_2(t) - i_3(t) + \frac{1}{C} v_2(t) \\
\frac{di_n(t)}{dt} & = i_n(t) - i_{n-1}(t) + \frac{1}{C} v_{n-1}(t)
\end{align*}
\]

\[
\frac{dv_1(t)}{dt} = \frac{1}{C} i_1(t) - \frac{1}{C} i_2(t) - \frac{G}{C} v_1(t)
\]

\[
\frac{dv_2(t)}{dt} = \frac{1}{C} i_2(t) - \frac{1}{C} i_3(t) - \frac{G}{C} v_2(t)
\]

\[
\frac{dv_n(t)}{dt} = \frac{1}{C} i_n(t) - \frac{1}{C} i_{n-1}(t) - \frac{G}{C} v_{n-1}(t)
\]
Main Advantages:

• Simulations direct in the time domain, without the use of inverse transforms and convolutions.
• Possibility of insertion and modeling of other time-variable power devices directly in the time domain (loads, protection devices, surge arresters, capacitors, etc).
• Detailed profile of voltages and currents at any point of the line.
• The line lumped parameters are set as a function of the electromagnetic transient considered as the input signal. This way good results can be obtained without the explicit representation of frequency-variable parameters.
Three-Phase Representation:

- A schematic representation of a three-phase system with its self and mutual parameters is given in the illustration.
- The line parameters are expressed as follows:

\[
[Z] = \begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix} \quad \quad [Y] = \begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
Y_{31} & Y_{32} & Y_{33}
\end{bmatrix}
\]
Three-Phase Representation:

- The modeling of the mutual parameters and coupling between phases is not a trivial task. This way modal analysis techniques are widely applied to model three-phase systems.

\[
[Z_m] = \begin{bmatrix} Z_{\alpha} & 0 & 0 \\ 0 & Z_{\beta} & 0 \\ 0 & 0 & Z_{0} \end{bmatrix}
\]

\[
[Y_m] = \begin{bmatrix} Y_{\alpha} & 0 & 0 \\ 0 & Y_{\beta} & 0 \\ 0 & 0 & Y_{0} \end{bmatrix}
\]

Where \([T_V]\) and \([T_I]\) are modal transformation matrices obtained from the eigenvectors and eigenvalues of the products \([Z][Y]\) and \([Y][Z]\) respectively.
Conventional Algorithm:

Line decoupling into their exact propagation modes.

Modeling of each exact mode by lumped elements.

Simulation of the currents and voltages in the Modal domain.

Conversion of modal values to the phase domain.
New Algorithm:

Line decoupling into their exact propagation modes.

Modeling of each exact mode by lumped elements.

Simulation of the currents and voltages in the Modal domain.

Conversion of modal values to the phase domain.

Proposal of a line model direct in the phase and time domains without the successive modal transformations.

Development direct in the phase domain.
Proposal of a Three-Phase Model in the Phase and Time Domains:

1) The modal parameters $R_m$, $L_m$, $C_m$, $G_m$ and the modal currents and voltages are expressed as functions of the phase parameters:

$$
[Z_m] = [T][Z][T]^T
$$
$$
[I_m] = [T][I]^{-1}
$$
$$
[Y_m] = [T][Y][T]^T
$$
$$
[V_m] = [T][V]^{-1}
$$

2) The parameters, currents and voltages in the phase domain are substituted in the differential equations of the line:

$$
\begin{align*}
\frac{dV_{h}}{dt} &= 2i_{h}(t) \cdot \frac{G}{CC} \\
\frac{di_{h}}{dt} &= 2i_{h}(t) \cdot \frac{11}{mnn} 
\end{align*}
$$
Proposal of a Three-Phase Model in the Phase and Time Domains:

3) The differential equations are represented in the state space:

\[ \dot{X} = AX + Bu(t) \]

- \( n \) is the number of pi elements.
- \( A, B \) and \( C \) are the phases.
- The elements of the state matrices \([A]\) and \([B]\) are function of the line parameters.
Proposal of a Three-Phase Model in the Phase and Time Domains:

- From the same way, other elements of matrices [A] and [B] are determined based on the self and mutual parameters of the line.

- The modal transformations are intrinsic considered in the space-state model.
Simulations:

- A conventional 440-kV transmission line is considered and a simple simulation test is performed based on the following system configurations:
Simulations:

- ATP/EMTP cascade of pi circuits: red-dash curve
- Proposed Model: blue curve
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- ATP/EMTP cascade of pi circuits: red-dash curve
- Proposed Model: blue curve
Conclusions:

• Results show a good accuracy when compared with those obtained from the cascade of pi circuits available in the ATP/EMTP.

• The three-phase model is entirely developed in the phase domain, without the explicit use of modal transformation matrices.

• Development of a three-phase representation and coupling between phases by a state space model, which represents the original proposal of the paper.
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