Modelling of Multi-Terminal HVDC Systems in Optimal Power Flow Formulation

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Multi-terminal HVDC (MTDC) systems

• Concern about the restricted power exchange due to lack of a strong interconnection between the countries within EU.

• Necessity of improving the level of power exchange as a result of development of the renewable energies.

• Multi-terminal HVDC (MTDC) systems:
  ✓ one the cost efficient ways to aggregate a huge amount of energy through interconnection of several renewable energy sources
  ✓ Connect the aggregated power to the existing AC systems through a common DC network

• MTDC systems can also be used for building an embedded DC grid in the large AC grids
Suppergrid Offshore Proposal
Steady State Modeling of the MTDC in the Existing AC Systems

- Extensive research to reveal steady state and dynamic behavior of such hybrid AC-DC grids.

- This study focuses on the modeling of VSC-based MTDC systems in the optimal power flow formulation.
AC Grid with Embedded MTDC System

AC System

DC Network

P_{CONV1} → Q_{CONV1} → P_{DC1}

PCC_1

Z_{eq1}

P_{CONVs} → Q_{CONVs} → ··· → P_{DCs}

PCC_s

Z_{eqs}

P_{DCs+1} ← P_{CONV_{s+1}} ← Q_{CONV_{s+1}}

PCC_{s+1}

Z_{eq_{s+1}}

V_{DC1} ← ··· ← V_{DCs} ← Z_{eqN} ← V_{DCN}

P_{DCN}

P_{CONVN} → Q_{CONVN} → PCC_N

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Steady State Model of VSC Station
AC and DC Sides Operating Modes

AC side control modes:

Active and reactive power control mode

Active and AC voltage control mode
AC and DC Sides Operating Modes

DC side control modes:

- **DC voltage droop mode**
- **Constant DC power mode**
- **Constant DC voltage mode**
AC and DC Sides Operating Modes

- DC slack bus:

One converter is considered as a DC slack converter to regulate its DC voltage around a specified value.

\[ P_{DC_1} + P_{DC_2} + \ldots + P_{DC_N} + P_{L,DC} = 0 \]
• $x$ is the vector of variables

\[
\begin{align*}
\text{Min} & \quad F(x) \\
\text{Subject to} & \quad H(x) = 0 \\
& \frac{G(x)}{} \leq G(x) \leq \overline{G(x)} \\
& \underline{x} \leq x \leq \overline{x}
\end{align*}
\]

• $F(x)$ is a scalar function of the vector $x$ known as objective function which can be fuel cost, active power losses or control components. In this paper the objective function of the optimal power flow formulation is the total cost of providing active powers.

• $H(x)$ is the equality constraint driven from the equations of combined AC and DC systems.

• $G(x)$ is a vector containing inequality constraints such as power transfer limit through the AC and DC lines.
AC Grid Equations

• Equality Constraints:

\[ 0 = P_{GD_i} - P_{INJ,AC_i} \]
\[ 0 = Q_{GD_i} - Q_{INJ,AC_i}, PQ \]
\[ 0 = V_i - V_{set_i}, PV \]

\[ P_{INJ,AC_i} = -V_i \sum_{m=1}^{M} V_m (G_{im} \cos(\theta_{im}) + B_{im} \sin(\theta_{im})) \]

\[ Q_{INJ,AC_i} = V_i \sum_{m=1}^{M} V_m (G_{im} \sin(\theta_{im}) + B_{im} \cos(\theta_{im})) \]

• AC state variables can be defined:

\[ X_{AC} = [V_{AC}^T, \theta^T, P_G^T, Q_G^T]^T \]
• Inequality Constraints:
The boundary conditions on
- nodal voltages
- generator active and reactive powers
- powers passing through the AC lines

\[
\begin{align*}
V_{AC_i} & \leq V_{AC_i} \leq \overline{V_{AC_i}} \\
\theta_i & \leq \theta_i \leq \overline{\theta_i} \\
P_{G_i} & \leq P_{G_i} \leq \overline{P_{G_i}} \\
Q_{G_i} & \leq Q_{G_i} \leq \overline{Q_{G_i}} \\
P_{Line_{i,j}} & \leq P_{Line_{i,j}} \leq \overline{P_{Line_{i,j}}}
\end{align*}
\]
Converters variables

- The mismatch equations applying to PCC buses are as follows:

  \[ 0 = P_{GD_i} - P_{INJ,AC_i} - P_{CONV_i} \]
  \[ 0 = Q_{GD_i} - Q_{INJ,AC_i} - Q_{CONV_i}, \text{ PCC as PQ bus} \]
  \[ 0 = V_i - V_{set_i}, \text{ PCC as PV bus} \]

- \( P_{CONV_i} \) and \( Q_{CONV_i} \) are converter powers at PCCs and are set to zero for non-PCC buses.

- Moreover, \( Q_{CONV_i} \) of the PCC bus whose converter is in the PV control mode is set to zero.

- Converter variables:

  \[ X_C = [P_{CONV}^T, Q_{CONV}^T]^T \]
DC Grid Equations

- **Equality Constraints:**
- The DC mismatch equations:

\[ 0 = P_{INJ,DC_i} - P_{DC_i} \]

where

\[ P_{INJ,DC_i} = -V_{DC_i} \sum_{n=1}^{N} V_{DC_n} G_{dc_{in}} \quad (i \neq s) \]

- **DC state variables** can be defined as follows:

\[ X_{DC} = V_{DC} = [V_{DC_1}, \ldots, V_{DC_{s-1}}, V_{DC_{s+1}}, \ldots, V_{DC_N}]^T \]

- **Inequality Constraints:**

\[ V_{DC_i} \leq V_{DC_i} \leq V_{DC_i} \]

\[ P_{Line,DC_{i,j}} \leq P_{Line,DC_{i,j}} \leq P_{Line,DC_{i,j}} \]
Slack Station Equation

- \( P_{\text{CONVs}} \) power at the PCC connected to slack converter is determined based on the DC network losses and other converters’ powers:

\[
P_{\text{CONVs}} = - \sum_{i=1, i \neq s}^{N} P_{\text{CONV}_i} - \sum_{i=1}^{N} P_{L,\text{station}_i} - P_{L,DC}
\]

- \( P_{L,\text{station}_i} \) is the total loss in each converter station which is a function of AC variables

- \( P_{L,DC} \), DC network losses, is a function of DC variables

- Therefore, \( P_{\text{CONVs}} \) is obtained based on DC, AC and converter variables (\( X_{\text{DC}}, X_{\text{AC}} \) and \( X_{C} \)).
The whole AC-DC Equations

\[ X = [V_{AC}^T, \theta^T, P_G^T, Q_G^T, P_{CONV}^T, Q_{CONV}^T, V_{DC}^T]^T \]

\[ H(x), \]

for non-PCC buses:
\[
0 = P_{GD_i} - P_{INJ,AC_i} \\
0 = Q_{GD_i} - Q_{INJ,AC_i}, PQ \text{ bus} \\
0 = V_i - V_{set_i}, PV \text{ bus}
\]

for non-slack PCC buses:
\[
0 = P_{GD_i} - P_{INJ,AC_i} - P_{CONV_i} \\
0 = Q_{GD_i} - Q_{INJ,AC_i} - Q_{CONV_i}, PQ \\
0 = V_i - V_{set_i}, PV
\]

for slack converter:
\[
0 = P_{CONV_s} + \sum_{i=1, i \neq s}^{N} P_{CONV_i} \\
+ \sum_{i=1}^{N} P_{L,\text{station}_i}(X_{AC}) \\
+ P_{L,\text{DC}}(X_{DC})
\]

for DC buses:
\[
0 = P_{INJ,DC_i} - P_{DC_i}
\]

\[ G(x), \]

\[
\frac{V_{AC_i}}{P_{G_i}} \leq V_{AC_i} \leq \frac{V_{AC_i}}{P_{G_i}} \\
\frac{\theta_i}{\bar{\theta}_i} \leq \theta_i \leq \frac{\theta_i}{\bar{\theta}_i} \\
P_{G_i} \leq P_{G_i} \leq \frac{P_{G_i}}{P_{G_i}} \\
Q_{G_i} \leq Q_{G_i} \leq \frac{Q_{G_i}}{Q_{G_i}} \\
P_{\text{Line},ij} \leq P_{\text{Line},ij} \leq \frac{P_{\text{Line},ij}}{P_{\text{Line},ij}} \\
P_{\text{CONV},i} \leq P_{\text{CONV},i} \leq \frac{P_{\text{CONV},i}}{P_{\text{CONV},i}} \\
Q_{\text{CONV},i} \leq Q_{\text{CONV},i} \leq \frac{Q_{\text{CONV},i}}{Q_{\text{CONV},i}} \\
P_{\text{CONV},s} \leq P_{\text{CONV},s} \leq \frac{P_{\text{CONV},s}}{P_{\text{CONV},s}} \\
V_{DC_i} \leq V_{DC_i} \leq \frac{V_{DC_i}}{V_{DC_i}} \\
P_{\text{Line,DC},ij} \leq P_{\text{Line,DC},ij} \leq \frac{P_{\text{Line,DC},ij}}{P_{\text{Line,DC},ij}}
\]
Case Study
# Simulation Results

## OPF Results Without MTDC

<table>
<thead>
<tr>
<th>AC Bus</th>
<th>Voltage (p.u.)</th>
<th>Phase (rad)</th>
<th>$P_{G_i}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0</td>
<td>44.730</td>
</tr>
<tr>
<td>2</td>
<td>1.003</td>
<td>-0.015</td>
<td>58.263</td>
</tr>
<tr>
<td>7</td>
<td>0.995</td>
<td>-0.070</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>1.045</td>
<td>-0.064</td>
<td>15.784</td>
</tr>
<tr>
<td>17</td>
<td>0.999</td>
<td>-0.102</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>0.974</td>
<td>-0.088</td>
<td>22.314</td>
</tr>
<tr>
<td>23</td>
<td>0.986</td>
<td>-0.073</td>
<td>15.784</td>
</tr>
<tr>
<td>27</td>
<td>1.050</td>
<td>-0.027</td>
<td>32.326</td>
</tr>
</tbody>
</table>

**Total Cost:** 565,2060 ($)

## OPF Results With MTDC

<table>
<thead>
<tr>
<th>AC Bus</th>
<th>Voltage (p.u.)</th>
<th>Phase (rad)</th>
<th>$P_{G_i}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0</td>
<td>44.883</td>
</tr>
<tr>
<td>2</td>
<td>0.997</td>
<td>-0.011</td>
<td>58.437</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>-0.049</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>1.050</td>
<td>-0.053</td>
<td>15.906</td>
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<tr>
<td>17</td>
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<td>-0.088</td>
<td>0</td>
</tr>
<tr>
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<td>-0.075</td>
<td>22.362</td>
</tr>
<tr>
<td>23</td>
<td>1.050</td>
<td>-0.062</td>
<td>15.906</td>
</tr>
<tr>
<td>27</td>
<td>1.007</td>
<td>-0.013</td>
<td>32.692</td>
</tr>
</tbody>
</table>

**Total Cost:** 568,9451 ($)
# Simulation Results

## OPF Results for MTDC System

<table>
<thead>
<tr>
<th>DC Bus</th>
<th>Voltage (p.u.)</th>
<th>$P_{CONV_i}$ (MW)</th>
<th>$Q_{CONV_i}$ (MVar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>1.050</td>
<td>10.000</td>
<td>6.068</td>
</tr>
<tr>
<td>D2</td>
<td>1.024</td>
<td>5.000</td>
<td>5.543</td>
</tr>
<tr>
<td>D3</td>
<td>0.993</td>
<td>14.014</td>
<td>5.000</td>
</tr>
</tbody>
</table>

## Voltage Profile Index and Total Cost in Percentage with and Without MTDC

<table>
<thead>
<tr>
<th></th>
<th>Without MTDC</th>
<th>With MTDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>VPI</td>
<td>1.3 %</td>
<td>2.4 %</td>
</tr>
<tr>
<td>Total Cost Change</td>
<td>0 %</td>
<td>0.6 %</td>
</tr>
</tbody>
</table>
CONCLUSION

- This paper presents an optimal power flow formulation for hybrid AC-DC networks.

- The constraints deviled into three groups of equations: (a) AC grid constraints, (b) multi-terminal HVDC constraints, and (c) DC grid constraints.

- The formulated AC-DC OPF is coded in GAMS platform and tested on IEEE 30 Bus system. Two scenarios of with and without MTDC system are studied and compared.

- The AC-DC OPF results from the system with MTDC shows better voltage profile as compared to the one without the MTDC. However, the total generation operating cost in the with-MTDC case is slightly increased.

- Further research is currently ongoing to give us more insight to the problem. The issue of locating the global optimum is also under research.
Thanks for your attention